and hence these scaling relationships satisfy the conditions for similitude.

Conclusions and Summary

Scaling relationships for the various characteristic parameters of the rapid motion of objects in ionized media pervaded by a magnetic field are obtained. It is seen that, if we relate all characteristic lengths (e.g., size of the object, particle mean free paths, cyclotron radii, etc.) in the laboratory experiment to the corresponding lengths in the actual flight situation by a scaling factor $\alpha = l_{mo}/l \ll 1$, then the pressure, density, magnetic field, and cyclotron frequency must equal $1/\alpha$ times their corresponding values in the actual case, provided that the mean flow speed, chemical composition, kinetic temperature, and degree of ionization are identical in the two situations. With these scaling relationships for the various parameters, the Mach number, the Knudsen number, the Reynolds number, the Hartmann number, and the ratios (H/ρ) and (coulomb drag/aerodynamic drag) are maintained identically equal in the model and the prototype, thereby satisfying the conditions of dimensional similitude. The Alfvén speed and plasma frequency are higher in the laboratory by a factor $\alpha^{-1/2}$ whereas the Debye length is lower by a factor of $\alpha^{1/2}$. These relationships are also in agreement because their product equals the speed of sound, which has identically equal values in the two situations.

References

[1] De Hoffman, F. and Teller, E., "Magnetohydrodynamic shock waves," Phys. Rev. **80**, 692 (1950).

[2] Chopra, K. P., "Some problems in hydromagnetics," Univ. of Southern California Engineering Center TN56-205, Air Force Office of Scientific Research AFOSR-TN-59-265 (1959).

[3] Chopra, K. P., "Interactions of rapidly moving objects in terrestrial atmosphere," Rev. Mod. Phys. **33**, 153 (1961).

Propellant Potential of Vaporized Metals in Temperature-Limited Rocket Systems

I. Glassman,* R. F. Sawyer,† and A. M. Mellor‡
Princeton University, Princeton, N. J.

In temperature-limited rocket propulsion systems such as electrothermal jet engines, solid core nuclear rockets, radioisotope rockets (Poodle), etc., it is accepted fundamentally that hydrogen is the best propellant because of its high enthalpy per unit mass. However, if one considers the latent heat of vaporization (condensation, sublimation) as another thermal degree of freedom, then it can be shown theoretically that the combination§ of the light metallic elements with hydrogen can surpass hydrogen in performance. In particular, it can be established that under space conditions the storable compound pentaborane is superior to cryogenic hydrogen. The result is possible because of the high heat of vaporization of boron, 11.2 kcal/g.

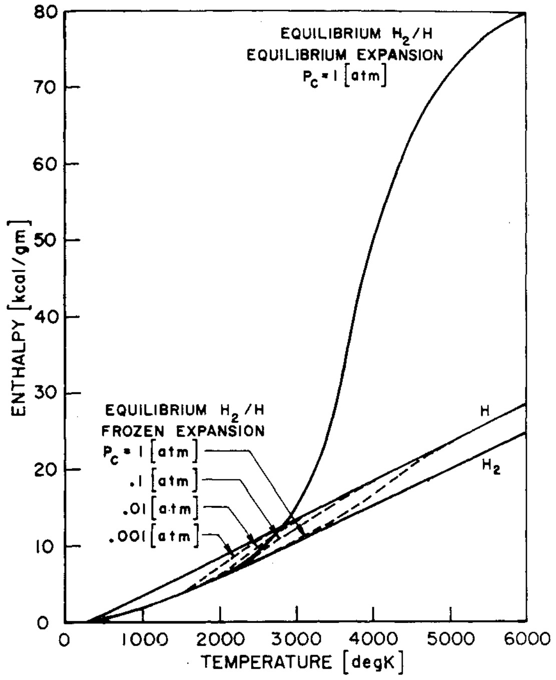

Fig. 1 Hydrogen enthalpies. Equilibrium[2] and frozen[3] expansion to 298°K.

Jack[1] was the first to consider the possible use of the light metallic elements as propellants in electrothermal jet engines. However, he restricted himself to a single metallic element as a working fluid. Dealing mainly with lithium because of its low atomic weight, he expressed concern whether condensation was desirable because of the formation of condensation shocks and the lack of a working fluid. The approach offered here differs from that of Jack in that the light metallic elements always are used in association with hydrogen as a carrier fluid. More important, though, it is argued that condensation is desirable and will take place some time during the nozzle expansion process. This assumption seems valid when one realizes that a complex condensed oxide like Al_2O_3 can be formed from suboxides and oxygen in rocket motors whose chamber temperatures are substantially higher than those found in temperature-limited systems.

The major assumptions and points are developed by describing the two figures. In Fig. 1 the available enthalpy per gram of various possible hydrogen systems is plotted. The square root of this enthalpy is, of course, directly proportional to the specific impulse. The *S*-shaped curve describes the available enthalpy for a hydrogen system heated to the temperature designated on the ordinate. The equilibrium dissociation of hydrogen is calculated at 1-atm total pressure. The H_2/H mixture obtained is expanded to the thermodynamic base temperature 298°K, and equilibrium is considered to be held throughout the expansion. Since the enthalpy at 298°K is the exhaust enthalpy, the curves represent the enthalpy available for propulsive work. The curves labeled H and H_2 are the sensible enthalpy of hydrogen atoms and molecules, respectively. The dotted lines represent the available enthalpy for equilibrium dissociation of H_2 and frozen expansion flows. From the kinetic recombination rates of H atoms, it appears that high-temperature H atom nozzle flows are essentially frozen. Consequently, comparisons here will be made with the lower set of curves in Fig. 1, and the dashed ones in particular.

Received August 13, 1964. The concept reported here was developed while work was being performed under metal combustion studies supported by NASA Grant NsG641 and chemical kinetic studies supported by Air Force Office of Scientific Research Contract AF49(638)-1268.

* Professor, Department of Aerospace and Mechanical Sciences, Guggenheim Laboratories for the Aerospace Propulsion Sciences. Associate Fellow Member AIAA.

† Assistant-in-Research, Department of Aerospace and Mechanical Sciences, Guggenheim Laboratories for the Aerospace Propulsion Sciences. Member AIAA.

‡ Assistant-in-Research, Department of Aerospace and Mechanical Sciences, Guggenheim Laboratories for the Aerospace Propulsion Sciences. Student Member AIAA.

§ The word combination is used here to mean a liquid hydride, any dispersion of metal particles in liquid hydrogen, liquid hydrogen and a gel containing metal particles, etc.

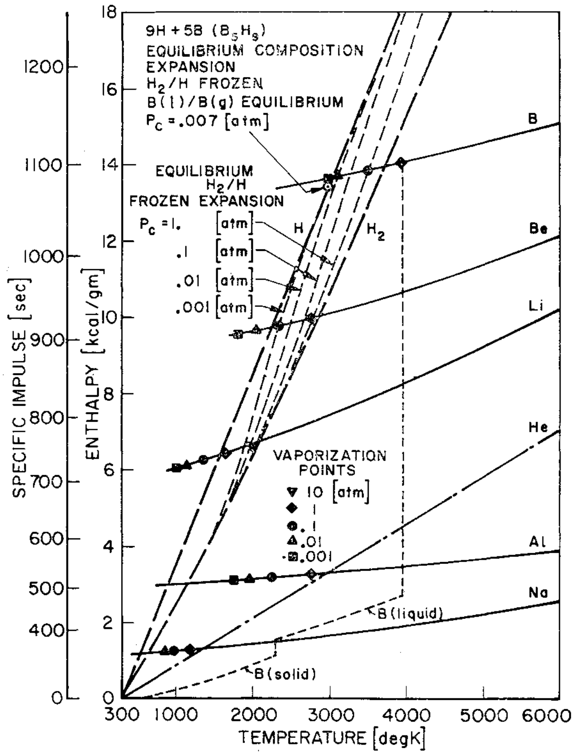

Fig. 2 Theoretical propellant performance of metallic vapors. Enthalpies from Ref. 3 except B and He from Ref. 4. Vapor pressures from Ref. 5.

In Fig. 2, these hydrogen data are replotted with curves for metallic vapors¶ and helium. All curves represent the sensible plus vaporization enthalpy above the base temperature. The temperatures at which the metallic element lines cross the H_2/H curves are the temperatures below which the metal system will be better than the pure hydrogen system, provided the metal can be vaporized below this temperature.

The specific impulse values listed on the ordinate scale are simply another interpretation of the enthalpy and have no physical significance for the vapor curves. For a practical case, one must introduce hydrogen as a carrier fluid. In this case the value of the enthalpy falls between the H_2/H curve and the vapor curve on the vertical at the temperature of operation. The impulse values now have meaning and represent an ideal case of complete condensation of the metallic vapor in the nozzle and no two-phase losses.

The major limitation of the metallized systems is the chamber pressure of operation. To obtain full benefit, the metal must be a vapor at the chamber temperature. Most metals have high normal boiling points and thus require low operating pressures. This statement is illustrated on Fig. 2 by the vaporization points plotted. For Be and Li, the points where the curves cross the H_2/H curve specify 1) the temperatures below which a gain in performance is obtained with addition of the metallic element and 2) the maximum operating pressures. However, for the higher atomic weight elements (B, Al, Na) with their higher normal boiling points, one can recognize readily that benefit from the metal vaporization can be obtained only at low pressures. The boron vapor curve crosses the 0.01-atm H_2/H curve at 3100°K. The vapor pressure of boron at this temperature is also about 0.01 atm. Thus, for equal pressures, boron containing systems will have higher theoretical performances than pure hydrogen systems at temperatures less than 3100°K. If a temperature limitation of 2900°K is imposed, the operating pressure becomes of the order of 0.001 atm, but the gain in impulse can be at least 40 sec (from 1045 to about 1085 sec).

The performance gains and limiting temperatures and pressures of operation for the other metallic species can be estimated from the information given in Fig. 2. The most realistic gains for practical high-temperature systems are obtained with boron. If a practical system demands a storable propellant like ammonia, then the metal comparisons should be made with such propellants and not hydrogen. The metals take on even greater promise under these conditions.

Further, it must be realized that, when the metallic elements are used with a carrier fluid, the vaporization points on Fig. 2 represent the partial pressures. Thus, the total pressures can be higher at a fixed temperature.

Pentaborane is particularly attractive for space systems limited to temperatures of 3000°K or below. Not only is it storable, but its specific impulse value for a temperature of 3000°K and a chamber pressure of 0.1 psia (0.0068 atm) is about 1087 sec, which is slightly higher than a pure hydrogen system. The value is plotted on Fig. 2 as the open point. The heated gas composition in mole fractions under these conditions is B (gas) = 0.385, H = 0.536, and H_2 = 0.079. The partial pressure of boron is about 0.0026 atm, and thus the operating pressure could be greater than the 0.0068 atm specified.

To establish whether the metallic vapor will condense before 298°K is reached in the nozzle, one must compare the partial pressure of the vapor at this exhaust temperature and the vapor pressure at 298°K. If the partial pressure is greater than the vapor pressure, condensation occurs. This comparison is readily made by considering the Clausius-Clapeyron equation and the logarithm of the isentropic pressure-temperature relationship:

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\Delta H_{\text{vap}}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right) \tag{1}$$

$$\ln\left(\frac{P_2}{P_1}\right) = \frac{\gamma}{\gamma - 1}\ln\left(\frac{T_2}{T_1}\right) \tag{2}$$

For ready comparison, P_1 is chosen as the vapor pressure and partial pressure of the metal at the chamber temperature T_1 and thus is the same in both equations. The chamber pressure would be P_1 divided by the mole fraction of metal vapor. In Eq. (1), P_2 is the vapor pressure at T_2 = 298°K, and in Eq. (2), P_2 is the partial pressure of the vapor in the flowing gas mixture at T_2 = 298°K. Thus, for condensation,

$$\frac{\gamma}{\gamma - 1}\left[\ln\frac{T_2}{T_1}\right] > \frac{\Delta H_{\text{vap}}}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right] \tag{3}$$

Since P_2 is lower than P_1, both terms are negative, and the relationship for condensation is written more appropriately as

$$\frac{\gamma}{\gamma - 1}\left[\ln\frac{T_1}{T_2}\right] < \frac{\Delta H_{\text{vap}}}{R\,T_1}\left[\frac{T_1}{T_2} - 1\right] \tag{4}$$

The value of $\gamma/(\gamma - 1)$ must be between 2.5 and 3.5. $\Delta H_{\text{vap}}/RT$ is simply Trouton's constant[6] divided by the universal gas constant R and is the order of 10. Further, if one expands $\ln(T_1/T_2)$ in a series, it immediately becomes obvious that $\ln(T_1/T_2)$ must be less than $[(T_1/T_2) - 1]$. Thus it is established that condensation always is favored thermodynamically in these metallized systems.

In conclusion, then, the proposed use of metal-hydrogen systems, as typified by pentaborane in temperature-limited systems, is theoretically higher in performance than a pure cryogenic hydrogen system in space applications and is thermodynamically possible.

¶ The condensed state enthalpy of boron is plotted on this figure as well.

References

[1] Jack, J. R., "Theoretical performance of propellants suitable for electrothermal jet engines," ARS J. **31**, 1685–1689 (1961).

[2] Zeleznik, F. J. and Gordon, S., "A general IBM 704 or 7090 computer program for computation of chemical equilibrium compositions, rocket performance, and Chapman-Jouget detonations," NASA TN D-1454 (October 1962).

[3] JANAF Thermochemical Tables, The Dow Chemical Co., Midland, Mich. (December 31, 1963).

[4] McBride, B. J., Heimel, S., Ehlers, J. G., and Gordon, S., "Thermodynamic properties to 6000°K for 210 substances involving the first 18 elements," NASA SP-3001, pp. 107, 196 (1963).

[5] Nesmeyanov, An. N., *Vapor Pressure of the Elements* (Academic Press, New York, 1963), pp. 456, 459, 460.

[6] Perry, J. H. (ed.), *Chemical Engineers' Handbook* (McGraw-Hill Book Co. Inc., New York, 1950), 3rd ed., p. 299.

Stagnation-Point Heat Transfer to Surfaces of Arbitrary Catalycity

HENRY TONG* AND BOB H. SUZUKI†

The Boeing Company, Seattle, Wash.

Nomenclature

C = atom mass fraction
f = dimensionless stream function [Eq. (6)]
h = static enthalpy
h^0 = dissociation enthalpy
H = nondimensional enthalpy
k_w = wall catalycity
L = nose radius
M_2 = molecular weight of molecules
p = pressure
Pr = Prandtl number
q = heating-transfer rate
R = universal gas constant
T = temperature
u_1 = tangential velocity gradient [Eq. (6)]
U = freestream velocity
v = normal velocity
w = mass production rate
y = normal coordinate
η = transform coordinate
μ_s = viscosity at shock wave
ρ = density
ρ_s = density at shock wave

Introduction

To the authors' knowledge no solution has been published for the viscous shock-layer equations using an arbitrary chemical reaction rate and an arbitrary surface catalytic efficiency. Since a glide re-entry vehicle spends much of its time in a flight regime where nonequilibrium conditions are likely to be important, it is desirable to delineate this regime and be able to predict the magnitude of the heat-transfer reduction. Large reductions in heating rates can result in significant reductions in vehicle weights which, in turn, reduce the power requirements for launch and control of the vehicle.

Analysis

To study the effects of chemical nonequilibrium on heat transfer, a binary air model is used. To simplify the problem it is assumed that the constituents of the air model are in thermal equilibrium with the exception of the electronic states, which are assumed to be unexcited. It is further assumed that the Lewis number, Prandtl number, and the product $\rho\mu$ are constant, and the flight conditions are restricted to the regime where the effects of ionization on heat transfer are negligible. The parameters for the gas model are obtained from a weighted average of the properties of oxygen and nitrogen. In particular, the equilibrium constant for recombination and dissociation is calculated by averaging the characteristic temperatures of dissociation, vibration, rotation (as defined by Lighthill), and the individual partition functions for the ground state of the electronic mode. The final equations are those given in Ref. 1 and are repeated below:

Momentum

$$f''' + 2ff'' + \left(\frac{\rho_\infty}{\rho} - f'^2\right) = 0 \tag{1}$$

Energy

$$H'' + 2PrfH' = 0 \tag{2}$$

Species

$$C'' + 2PrfC' = -\frac{Pr}{(2)^{1/2}}\left(\frac{L}{U}\right)\frac{w}{\rho} \tag{3}$$

State

$$p = \rho(RT/M_2)(1 + C) \tag{4}$$

Caloric

$$h = \left(\frac{9}{2} + \frac{1}{2}\,C\right)\frac{RT}{M_2} + h^0C \tag{5}$$

where

$$\begin{aligned} \eta &= \left[\frac{2^{1/2}U}{\rho_s\mu_sL}\right]^{1/2}\int_0^y \rho dy \\ v &= -2^{3/2}\left(\frac{U}{L}\right)\frac{dy}{d\eta}\,f \\ u_1 &= 2^{1/2}\left(\frac{U}{L}\right)f' \end{aligned} \tag{6}$$

The boundary conditions for the foregoing equations are:

At $\eta = 0$

$$\begin{aligned} &f_w = f_w' = 0 \qquad C_w' = \left(\frac{\rho_s\mu_sL}{2^{1/2}U}\right)\left(\frac{k}{\rho D}\right)_w C_w \\ &H_w = \left(\frac{2}{U^2}\right)\left[\left(\frac{9}{2} + \frac{1}{2}\,C_w\right)\frac{RT}{M_2} + h^0C_w\right] \end{aligned} \tag{7}$$

At $\eta = \eta_s$

$$\begin{aligned} &f_s = 2^{-5/4}\left(\frac{UL}{\rho_s\mu_s}\right)^{1/2}\rho_\infty \qquad f_s' = 2^{-1/2} \\ &H_s = 1 \qquad C_s = 0 \end{aligned} \tag{8}$$

The properties behind the shock wave are obtained by solving the Rankine-Hugoniot equations using the hypersonic flow simplifications and assuming that the atom concentration is zero immediately behind the shock wave.

Results

Some results for a nose radius of 1 ft are shown in Figs. 1–3. Since it is not possible to include all the results in this note, the interested reader is referred to Ref. 2. Figure 1 shows the variation of heat transfer as a function of the freestream velocity at a fixed altitude of 230,000 ft and wall catalycities of zero and infinity. At a velocity of 15,000 fps

---

Received July 2, 1964.

* Research Engineer.

† Research Engineer; now Graduate Student at the California Institute of Technology, Pasadena, Calif.